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FREE ACTIONS ON REVERSE SEMIDERVATIONS

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ABSTRACT

The study of Laradji and Taheem in [5] inspired us and in this paper we authors introduce the notion of dependent elements of reverse semiderivations and generalized reverse semiderivations on semiprimesemirings. Also in this paper we authors study and investigate dependent elements of reverse semiderivations and generalized reverse semiderivations and proved that reverse semiderivations, generalized reverse semiderivations and related mappings on semiprimesemirings are free actions.

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I. INTRODUCTION

The study of semiring dates back to H.S.Vandiver[13].Throughout this paper S will represent a commutativesemiring.The concept of free action was introduced by Murray and von Neumann [6] and von Neumann for commutative von Neumann algebras[10]. In [11] Josovukman and Irena KosiUlbl worked on dependent elements of derivations on rings. Dependent elements were implicitly used by Kallman [4] to extend the notion of free action of automorphisms of abelian von Neumann algebras of Murray and von Neumann. They were later on introduced by Choda et al. [1]. Several other authors have studied dependent elements in operator algebras. Recently, Vukman and Kosii-Ulble[12] further explored dependent elements of certain mappings on prime and semiprime rings. A brief account of dependent elements in W∗ -algebras has been also appeared in the book of Stratila [9]. In [7]R.Murugesan et al introduced dependent elements on semiderivations and proved that semiderivations and related mappings are free actions. Motivated by all these we authors in this paper investigated dependent elements of some mappings related to reverse semiderivations and generalized reversesemiderivations on semiprimesemirings. In this paper we characterize dependent elements of reverse semiderivations and generalized reverse semiderivationson semiprimesemirings and also we study the reverse semiderivations, generalized reverse semiderivations and related mappings on a semiprimesemiring S are free actions.

Definition 1.1. A semiring S is a nonempty set S Equipped with two binary operations + and · such that
(i). (S, +) is a commutative monoid with identity element 0
(ii). (S, ·) is a monoid with identity element 1
(iii). Multiplication left and right distributes over addition.

A commutative semiring is one whose multiplication· is commutative.
Definition 1.2. A semiring S is said to be prime if $xSy = 0$ implies $x = 0$ or $y = 0$ for all $x, y \in S$

Definition 1.3. A semiring S is said to be semiprime if xsx = 0 implies x = 0 for all x ∈ S
Definition 1.4. An additive mapping d : S → S is called a derivation if d(xy) = d(x)y + xd(y) holds for all x, y ∈ S
Definition 1.5. An additive mapping D : S → S is called a generalized derivation associated with the derivation d : S → S if D(xy) = D(x)y + xd(y) holds for all x, y ∈ S
Definition 1.6. An additive mapping f : S → S is called a semiderivation associated with a function g : S → S if for all x, y ∈ S
(i). f(xy) = f(x)g(y) + xf(y) = f(x)y + g(x)f(y), (ii). f(g(x)) = g(f(x)).
If g = I i.e., an identity mapping of S then all semiderivations associated with g are merely ordinary derivations. If g is any endomorphism of S, then semiderivations are of the form f(x) = x − g(x).
Definition 1.7. An additive mapping F : S → S is called a generalizedsemiderivation associated with a semiderivationf : S → S if for all x, y ∈ S
(i). F(xy) = F(x)y +g(x)f(y) = f(x)g(y) + xF(y), (ii). F(g(x)) = g(F(x)).
Definition 1.8. An additive mapping r : S → S is called a reversesemiderivation associated with a function g : S → S if for all x, y ∈ S
(i). r(xy) = r(y)g(x) + yr(x) = r(y)x + g(y)r(x), (ii). r(g(x)) = g(r(x)).
Definition 1.9. An additive mapping R : S → S is called a generalized reversesemiderivation associated with a reverse semiderivationr : S → S if for all x, y ∈ S
(i). R(xy) = R(y)x +g(y)r(x) = r(y)g(x) + yR(x), (ii). R(g(x)) = g(R(x)).
Definition 1.10. Let S be a semiprimesemiring. An element a in S is called a dependent element on a mapping f : S → S if f(x)a = ax for all x ∈ S
Definition 1.11. A mapping F : S → S is called a free action in case zero is the only element dependent on F.
We say that cancellation laws hold in a semiring S if
$ab=bc(a\neq 0)\Rightarrow b=c$ and $ba=ca(a\neq 0)\Rightarrow b=c$ a,b,c are in S.

II. RESULTS

Theorem 2.1
Let S be a commutative semiprimesemiring. Let $r: S \to S$ be a reverse semiderivation associated with a function $g: S \to S$. Then r is a free action.
Proof:
Let a in S be a dependent element of r. Then for all x in S and a in S we have
$r(x)a = ax$ (1)
Replacing xy for x in (1)
$r(xy)a = axy$, for all x, y in S
$r(y)g(x)a + yr(x)a = axy$, for all x, y in S
$r(y)g(x)a + yax = axy$ by (1), for all x, y in S (2)
Replacing xz for x in (2)
$r(y)g(xz)a + yaxz = axzy$, for all x, y, z in S
$r(y)g(xz)a + yaxz = axyz$, since S is commutativeand, for all x, y, z in S (3)
Right multiplying (2) by z
$r(y)g(x)az + yaxz = axyz$, for all x, y, z in S (4)
Equating (3) and (4)
$r(y)g(xz)a = r(y)g(x)az$, for all x, y, z in S
By left cancellation law,
$g(xz)a = g(x)az$, for all x, z in S
$g(x)g(z)a = g(x)az$, since g is a homomorphism and for all x, z in S

$g(z)a = az$, for all z in S

$za = az$, since g is surjective. (5)

Now using (5) in (2),

$r(y)g(x)a + ayx = axy$, for all x, y in S

$r(y)g(x)a + ayx = ayx$ since S is commutative and for all x, y in S

$r(y)g(x)a = 0$, for all x, y in S

$r(y)xa = 0$ since g is surjective and for all x, y in S (6)

Replacing x for ax in (6),

$r(y)axa = 0$, for all x in S

$ayxa = 0$, for all x, y in S

$aSa = 0$

$a = 0$, since S is semiprime.

Hence r is a free action.

Theorem 2.2

Let S be a commutative semiprimesemiring. Let $R : S \to S$ be a generalized reverse semiderivation associated with the semiderivation $f : S \to S$. Then R is a free action.

Proof:

Let a in S be a dependent element of R and r Then for all x in S and a in S we have

$R(x)a = ax$ and $r(x) = ax$ (1)

Replacing xy for x in (1)

$R(xy)a = axy$, for all x, y in S

$r(y)g(x)a + yR(x)a = axy$, for all x, y in S

$r(y)g(x)a + yax = axy$ by (1), for all x, y in S (2)

Replacing xz for x in (2)

$r(y)g(xz)a + yaxz = axzy$, for all x, y, z in S

$r(y)g(xz)a + yaxz = axyz$, since S is commutative and, for all x, y, z in S (3)

Right multiplying (2) by z

$r(y)g(x)az + yaxz = axyz$, for all x, y, z in S (4)

Equating (3) and (4)

$r(y)g(xz)a = r(y)g(x)az$, for all x, y, z in S

By left cancellation law,

$g(xz)a = g(x)az$, for all x, z in S

$g(x)g(z)a = g(x)az$, since g is a homomorphism and for all x, z in S

$g(z)a = az$, for all z in S

$za = az$, since g is surjective. (5)

Now using (5) in (2),

$r(y)g(x)a + ayx = axy$, for all x, y in S

$r(y)g(x)a + ayx = ayx$ since S is commutative and for all x, y in S

$r(y)g(x)a = 0$, for all x, y in S

$r(y)xa = 0$ since g is surjective and for all x, y in S (6)

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Replacing x for ax in (6),

$r(y)axa = 0$, for all x in S

$ayxa = 0$, for all x, y in S

$aSa = 0$

$a = 0$, since S is semiprime.

Hence R is a free action.

Theorem 2.3

Let S be a commutative semiprimesemiring. Let $r: S \rightarrow S$ be a reverse semiderivation associated with a function $g: S \rightarrow S$. Then for all x in S, the mapping $\phi: x \rightarrow xr(x)$ is a free action.

Proof:

Given that $\phi(x) = xr(x)$ for all x in S

Let a in S be a dependent element of ϕ. Then for all x in S and a in S we have

$\phi(x)a = ax$ (1)

$xr(x)a = ax$ (2)

Linearizing (2) with respect to x

$xr(y)a + yr(x)a = 0$, for all x, y in S (3)

Replacing x and y by a in (3)

$ar(a)a + ar(a)a = 0$

$2ar(a)a = 0$ (4)

Replacing y by ax in (3)

$xr(ax)a + axr(x)a = 0$, for all x in S

$xr(x)g(a)a + xxr(a)a + axr(x)a = 0$, for all x in S

$xr(x)aa + xxr(a)a + axr(x)a = 0$, for all x in S

$axa + xxr(a)a + aar(x)a = 0$, for all x in S (5)

Replacing x by a in (5)

$aaa + 2aar(a)a = 0$

$aaa + a2ar(a)a = 0$

$a^3 = 0$, by (4)

$a = 0$, Hence ϕ is a free action.

Theorem 2.4

Let S be a commutative semiprimesemiring. Let $R: S \rightarrow S$ be a generalized reverse semiderivation associated with the semiderivation $f: S \rightarrow S$. Then for all x in S, the mapping $\phi: x \rightarrow xR(x)$ is a free action.

Proof:

Given that $\phi(x) = xR(x)$ for all x in S

Let a in S be a dependent element of ϕ. Then for all x in S and a in S we have

$\phi(x)a = ax$ (1)

$xR(x)a = ax$ for all x in S (2)

Linearizing (2) with respect to x

$xR(y)a + yR(x)a = 0$, for all x, y in S (3)

Replacing x and y by a in (3)

$aR(a)a + aR(a)a = 0$

$2aR(a)a = 0$ (4)

Replacing y by ax in (3)

$xR(ax)a + axR(x)a = 0$, for all x in S

$xR(x)aa + xg(x)r(a)a + axR(x)a = 0$, for all x in S (5)

Replacing x by a in (5)

$aR(a)aa + ag(a)r(a)a + aaR(a) = 0$

$2aR(a)aa + ag(a)r(a)a = 0$

$ag(a)r(a)a = 0$, by (4)

$asa = 0$, since $g(a)r(a) \in S$

$a = 0$ by the semiprimeness of S. Hence ϕ is a free action.

Theorem 2.5

Let S be a 2- torsion free semiprimesemiring. Let $r : S \to S$ be a reverse semiderivation associated with a function $g : S \to S$. Then for all x in S, the mapping $\phi : x \to xr(x) + r(x)x$ is a free action.

Proof:

Given that $\phi(x) = xr(x) + r(x)x$ for all x in S

Let a in S be a dependent element of ϕ. Then for all x in S and a in S we have

$\phi(x)a = ax$ (1)

$xr(x)a + r(x)xa = ax$ (2)

Linearizing (2) with respect to x

$xr(y)a + yr(x)a + r(x)ya + r(y)xa = 0$, for all x, y in S (3)

Replacing x and y by a in (3)

$ar(a)a + ar(a)a + r(a)aa + r(a)aa = 0$

$2(ar(a)a + r(a)aa) = 0$

Since S is 2- torsion free

$ar(a)a + r(a)aa = 0$ (4)

Replacing x by a in (2)

$ar(a)a + r(a)aa = a^2$ (5)

Using (4) in (5) we get $a^2 = 0$

Ie, $a = 0$ Hence ϕ is a free action.

Theorem 2.6

Let S be a 2- torsion free semiprimesemiring. Let $R : S \to S$ be a generalized reverse semiderivation associated with the semiderivation $r : S \to S$. Then for all x in S, the mapping $\phi : x \to xR(x) + R(x)x$ is a free action.

Proof:

Given that $\phi(x) = xR(x) + R(x)x$ for all x in S

Let a in S be a dependent element of ϕ. Then for all x in S and a in S we have

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$\phi(x)a = ax$ (1)

$xR(x)a + R(x)xa = ax$ for all x in S
(2)

Linearizing (2) with respect to x

$xR(y)a + yR(x)a + R(x)ya + R(y)xa = 0$, for all x, y in S (3)

Replacing x and y by a in (3)

$aR(a)a + aR(a)a + R(a)aa + R(a)aa = 0$

$2(aR(a)a + R(a)aa) = 0$

Since S is 2- torsion free

$aR(a)a + R(a)aa = 0$ (4)

Replacing x by a in (2)

$aR(a)a + R(a)aa = a^2$ (5)

Using (4) in (5) we get $a^2 = 0$

Ie, $a = 0$ Hence ϕ is a free action.

Theorem 2.7

Let S be a 2- torsion free semiprimesemiring. Let $r_1 : S \to S$ and $r_2 : S \to S$ be two reverse semiderivations associated with functions $g_1 : S \to S$ and $g_2 : S \to S$ respectievely. Then for all x in S, the mapping $\phi : x \to r_1(x) + r_2(x)$ is a free action.

Proof:

Given that $\phi(x) = r_1(x) + r_2(x)$ for all x in S

Let a in S be a dependent element of ϕ. Then for all x in S and a in S we have

$\phi(x)a = ax$ (1)

$r_1(x)a + r_2(x)a = ax$ (2)

Replacing x by yx in (2)

$r_1(yx)a + r_2(xy)a = ayx$ for all x, y in S

$r_1(x)g_1(y)a + xr_1(y)a + r_2(x)g_2(y)a + xr_2(y)a = ayx$ for all x, y in S

$[r_1(x) + r_2(x)]ya + x[r_1(y) + r_2(y)]a = ayx$

$\phi(x)ya + x\phi(y)a = ayx$ (3)

Replacing x by ya in (1)

$\phi(ya)a = aya$

Replacing x by a in (3)

$\phi(a)ya + a\phi(y)a = aya$

$aya + aay = aya$

$a^2 y = 0$ (4)

Replacing y by a in (4) we get $a^3 = 0$

Ie, $a = 0$ Hence ϕ is a free action.

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